

Discovering Price-Load Relationships in California's Electricity Market¹

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Abstract. This paper reports on characterizing recent price behavior in the California electricity market. Market participants, that is, producers, consumers and traders, are highly motivated by the potential for profits to develop strategies to explore, and exploit, the limits of system operation. These strategies should be reflected in the market as different price to load relationships. We show that a number of regimes, i.e., characteristic behaviors, exist in the price time series, and provide a brief analysis of each regime. Knowledge of the number of regimes, their characteristics and switching dynamics allows insight into the market and power system performance.

Keywords. Deregulation, Electricity market, Power industry, Power system economics, Time series, Price prediction.

I. INTRODUCTION

The fundamental objective of electric power industry deregulation is to provide a competitive business environment, which, over time, will maximize an overall societal good: efficient generation and consumption of electricity. It is obviously quite important to continually assess if the industry is truly moving towards such an end. Recent events, such as, the Midwest price volatility in the summer of 1998 and several major outages in the summer of 1999 [1], have raised doubts in the public eye about deregulation. The Federal government, while particularly anxious about system reliability, is also concerned that the new regulations do not lead to such problems as, price manipulation, or structural barriers to the introduction of new technologies. An important aspect of assessing the markets is to understand how price behavior reflects underlying system performance. This paper reports on characterizing recent price behavior in the California market.

The operation of the power market is far more involved than for traditional markets since the electric commodity must be generated, distributed and consumed in real-time under strict physical laws and extremely high reliability requirements. Speaking broadly, an electricity market uses some bidding scheme to provide a generation schedule, which a governing independent grid operator (IGO) evaluates in order to ensure viability. This evaluation involves extensive off-line studies (e.g., the WSCC criteria [2]) of both static and dynamic behavior arising from major contingencies. If the market schedules present security concerns, then adjustments must be made to the schedules with corresponding price changes. For example in California, a zonal pricing scheme is used and supplemental bids are then used to set price [3].

The physical system is extremely complex and today many power system phenomena are not fully understood. With the introduction of markets, and potentially rapidly changing generation schedules, analysis is yet more difficult. Still, market participants, that is, producers, consumers and traders, are highly motivated by the potential for profits to develop strategies to explore, and exploit, limits of system operation. This will be reflected in market behavior as pricing patterns. While there has been fundamental, and important, work on the strategies that market participants could pursue to take advantage of system limits [e.g., 4], or to take advantage of system instabilities [e.g., 5], such studies suffer from a disadvantage of trying to understand events from highly simplified models of the market, or of the power system, or of both. In contrast, the work in this paper uses price data directly in the belief that such analysis can indicate market participants' behavior as well as the underlying power system performance. Discovered behavior would require subsequent analysis to identify the underlying cause but still be useful for market participants in the same measures such as volatility are important.

Power system performance may be assessed from a number of perspectives. Under a traditional engineering viewpoint, such measurements as outage statistics, efficiency, security indices, and so on, all provide insight into the

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system capabilities. Conversely, the economics viewpoint may focus on whether electricity prices settle near the marginal cost of generation as classical demand-supply theories suggest. Both of these viewpoints are valid and need to be considered. Still, neither can capture the behavior of trades and various financial instruments, which reflect the health of a market. For example, the existence of predictable patterns of trades and price movements that reflect market inefficiency is of concern. More specifically, we are concerned with addressing the temporal aspects of financial markets, such as, price predictability and volatility, with respect to electricity trades. In this vein, this paper analyzes hourly-unconstrained zonal price data from April 1, 1998 – October 1, 1999 of the day-ahead California electricity market and forecasted load. We show that a number of regimes, i.e., characteristic behaviors, exist in the price time series, and provide a brief analysis of each regime. Knowledge of the number of regimes, their characteristics and switching dynamics allows insight into the market behavior and performance. These results can facilitate the discovery of possible market inefficiencies and may be useful in determining if regulatory modifications are needed. Furthermore, public knowledge of the existence of different pricing regimes may allow for more effective bidding by market participants.

II. BACKGROUND

In the California market, hourly-unconstrained market clearing prices (MCP) for the majority of the market are determined on day-ahead and hour-ahead spot markets, e.g., by the power exchange (PX) [3]. These prices reflect much more than just the costs of the suppliers or needs of the consumers. For example during off-peak hours, the price is often extremely low since suppliers may wish to avoid the cost of shutting down a unit while waiting for the higher priced peak hours. Unpredicted events, such as equipment outages, can also influence MCP. An important implication of this, for the study here, is that the price-load relationship is neither linear nor stationary in time. However, it is to be expected that price-load relationship is relatively stable over shorter periods of time. In contrast to load forecasting, where there are relatively well-understood load patterns (weekdays vs. holidays and weekends, unusually hot weather, and so on), volatile electricity prices in power markets are a new phenomenon that needs to be examined.

An issue in electricity price modeling can be the interdependence between price and load. In the econometrics literature, the problems involving prediction of mutually dependent variables often require solving systems of simultaneous equations [6]. As will be shown later, the characteristic shape of demand and supply curves of deregulated power market is characterized by nearly perfectly inelastic demand, i.e. by an almost vertical demand curve. This means that, at least as a first approximation, load is not influenced by electricity price. With this assumption, the price-load relationship can be considered as a standard regression problem, and solving simultaneous equations can be avoided.

If a time series of hourly MCP can be partitioned into intervals with stable price-load relationships, it is said to be a *piece-wise stationary* time series. All intervals with the same relationship are called a *regime*. One regression model should be learned separately on each regime to successfully represent all existing price-load relationships. The objective of this study is to search for possible pricing regimes in the electricity MCP, for example, periods with particularly high prices or other unusual price-load relationships. Since pricing regimes are not known in advance, the analysis consists of two steps: (1) discovering existing regimes and their positions within a time series, and (2) analyzing each of these regimes separately.

Some authors have offered possible solutions for this nontrivial problem. One of them is the gated experts approach [7] where a time-series is partitioned into a number of subsets and separate models are learned on each of them. The drawback of gated experts is that partition is based solely on input variables, making this method unable to discriminate between regimes if switching between regimes does not depend on available input variables (this is the most likely scenario for electricity markets). The other solution could be modeling the switching between regimes as a Markov process [8]. Drawbacks of this procedure are that the number of regimes must be pre-specified, and that there often exists little evidence to support the Markov process assumption. In the next section, a regime discovery algorithm overcoming these limitations is described.

A. Regime Discovery Algorithm

The proposed approach applies an algorithm capable of discovering regimes present in time-series without supervision using only the assumption that the rate of switching between regimes is not high. The method relies on competition between separate regression models for each part of a given time series. The regime discovery algorithm starts by learning a global regression model on the complete time series. Next, the time series is split into

two disjoint subsets of equal size and a separate regression model is trained for each of them. The two models then compete for each data point, such that, a given point is assigned to the model achieving a smaller error. This procedure iterates until a stable partitioning is obtained. Subsequently, the prediction error obtained by such partition is compared to the prediction error of the global model. If using two regression models is better than a simple global model, a third model is added to the competition in a similar fashion to the above. The procedure continues by adding new models into the competition until it has been concluded that each model is proficient for one of the existing regimes, and that further partitioning does not improve predictability of the time series.

Filtering absolute errors for each competing model over neighboring data points is very important in the proposed algorithm. The notion arises from the likely condition that in a time series with regimes, neighboring points are likely to stem from the same regime so one should try to prevent assigning neighboring points to different regimes. Averaging the absolute errors of each of L competing models over the neighboring l points should decrease the probability of a given data point being assigned to a wrong model. This approach will encounter problems if the switching rate between regimes is higher than half the filter length.

The regime discovery algorithm is formally described in Fig. 1 with the following notation. A data set representing a time series of length T is defined as

$$S_0 = \{(\mathbf{x}_t, y_t), t \in [1, T]\} \quad (1)$$

where $\{y_t\}$ is time series of MCP and \mathbf{x}_t is a vector consisting of lags of y_t and of the exogenous variables (e.g., forecasted load). If time interval $[1, T]$, is partitioned into L disjoint subsets T_i , $i=1, \dots, L$, the corresponding data subsets are denoted as S_i , where $S_i = \{(\mathbf{x}_t, y_t), t \in T_i\}$, and the whole partition as $\mathbb{S}_L = \{S_1, \dots, S_L\}$. Separate regression models, $M_i(\mathbf{x}, \boldsymbol{\beta})$, with $\boldsymbol{\beta}$ a vector of parameters, describe the relationships between y_t and \mathbf{x}_t , and are built on each of the subsets S_i , as

$$\hat{y}_t = M_i(\mathbf{x}_t, \boldsymbol{\beta}), \text{ if } t \in T_i. \quad (2)$$

where \hat{y}_t is the composite prediction and the *composite error* is defined as the mean squared error (MSE) of \hat{y}_t . For further reference to the regime discovery algorithm and its convergence properties, see [9].

B. Remarks

To apply the regime discovery algorithm, three design decisions must be made. The first decision involves the choice of input variables, \mathbf{x} , and the associated structure of regression models, $M(\mathbf{x}; \boldsymbol{\beta})$. One needs to allow for a fair approximation of the true underlying relationship, and thus, any prior knowledge about the data will be useful. Nonetheless, the proposed algorithm is not restrictive in the choice of regression model, but can be used over a range of linear and nonlinear models. The second design decision concerns the parameter α , which sets the stopping criterion. The closer α is to I , the lower error but the more regimes that will be allowed. Again, prior knowledge may provide useful feedback on the correct choice of this parameter. For example as will be seen in section 3, the discovered regimes roughly reflect seasonal behavior, while a value of α closer to I might improve accuracy slightly, it would come at the cost of insight to the problem. The third design decision governs the type and length of error filter. In our previous work [9], it has been shown that the algorithm is robust over a large range of filter lengths. Yet, if the switching between regimes is significantly faster than the filter length, the algorithm will not converge. Using median filters instead of moving average filters would make the algorithm additionally robust with regards to outliers, but moving average filters have been used here without any significant degradation of performance.

III. CASE STUDY OF CALIFORNIA MARKET

This section analyzes recent price behavior in the California market. Publicly available data on day-ahead forecasted load known before trading and MCP of day-ahead market in the period from April 1, 1998 till September 30, 1999, are used in the experiments. First, an attempt is made to model prices with a single regression model. This allows some insight into the relationship between price and load as well as identifying the important features of the time-series. Second, any hidden non-linearities not captured in the model are sought. If such non-linearity exists, more involved regression models will be needed. Finally, multiple regimes are allowed in the modeling to investigate the different price-load behaviors that existed during this time period.

A. Modeling Price-Load Relationship With a Single Model

1) Choice of Variables

The first step in MCP prediction is the choice of input variables and the functional form of the regression function. Among the publicly available data, the forecasted load, historical MCP, and functions of these variables, are clearly the most useful for prediction. Table 1 lists the features X_i used for the first experiment of determining important features, together with their correlation to MCP.

Prediction results are obtained by applying ordinary least squares (OLS) algorithm on all available data to fit the function of the form

$$Y_t = \beta_0 + \sum \beta_i X_{it} \quad (3)$$

where Y_t is the time series of hourly MCP. This is repeated for different subsets of the 15 available features and presented in Table 2. The coefficient of determination, R^2 , defined as $R^2 = 1 - MSE/\sigma^2$ where σ is price standard deviation, is used as a measure of predictive capabilities of different regression models.

As can be seen, information of load modeled as a third order polynomial and yesterday's MCP is adequate for successful MCP prediction. The large influence of yesterday's MCP on today's MCP is a good indicator that market behavior can be considered as relatively stable over shorter periods of time; however, it does not provide insight into the reasons for periods of extremely unusual prices.

2) Hidden Nonlinearities

Hidden nonlinearities not captured by linear functional form (3) are sought in the following experiment. Since neural networks [10] are known to be able to represent highly nonlinear relationships in the data, neural networks with five hidden nodes have been trained to predict the hourly MCP for 3 different sets of input variables, as shown in Table 2. As can be seen, no significant improvement is achieved by using neural networks over the simpler linear functional form. Further, the third order polynomial appears to be sufficient to capture the relationships between MCP and forecasted load.

3) Influence of Hour and Day on Price Prediction

In load forecasting, there are relatively well-understood load patterns (peak and off-peak hours, weekdays vs. holidays and weekends, unusually hot weather, and so on). This knowledge is critical for successful load forecasting. In the following experiments, the existence of price patterns with respect to hour or day is examined, and the results are compared to load forecasting. The prediction model (3) is estimated with input variables $\{X_1, X_2, X_3, X_{12}\}$ on the complete available data set. Fig. 2a and Fig. 2b show mean errors and one standard deviation away from of the 24 hour and 7 day predictions, respectively. It is obvious that the obtained prediction model does not show significant bias for any particular hour or day, which indicates that such information is not useful for price prediction. To illustrate day and hour influence on load forecasting, a prediction model of $l_t = \beta_1 l_{t-1} + \beta_2 l_{t-24} + \beta_3 l_{t-24*7}$, where l_t is load at time t , is also constructed. The model achieves $R^2 = 0.96$, with errors and deviations shown in Fig. 2c and Fig. 2d. Significant bias of the prediction exists both for particular hours and days, indicating that proper modeling must include information on hour and day. This highlights a difference between price forecasting and load forecasting and can be explained by the shape of supply-demand curve of the electricity market (Fig. 3).

For the current implementation of the deregulated market, many consumers are not aware of the high volatility of electricity price on the market (indeed are not even charged hourly prices) and so are not motivated to change their consumption behavior with price. As a consequence, electricity consumption is influenced more by the patterns of consumer's behavior, and very little by the current market price. Time of the day and day of the week are therefore very useful in modeling and prediction of load in power systems. On the other hand, the shape of the supply curve indicates stiff competition among generators for the right to supply electricity. That is, the competition among generators almost fully determines the MCP. Influences, such as, scheduled maintenance or the season, are likely to be significant, while the time of day and the day of the week are not relevant since they are already fully reflected in the forecasted load.

B. Discovering Multiple Regimes

The proposed regime discovery algorithm has been employed to discover possible pricing regimes in California's market. Experiments have been performed with two different sets of input variables: 1) $\{X_1, X_2, X_3\}$, and 2) $\{X_1, X_2, X_3, X_{12}\}$. In the first experiment, only forecasted load is used for price prediction, which allows visualization and

easier explanation of obtained results. Using Table 1, regression functions used for competition in experiment 1 can be expressed as $p_t = \beta_0 + \beta_1 l_t + \beta_2 l_t^2 + \beta_3 l_t^3$, where p_t is MCP and l_t is predicted load for hour t . In the second experiment, yesterday's MCP has been included, so $p_t = \beta_0 + \beta_1 l_t + \beta_2 l_t^2 + \beta_3 l_t^3 + \beta_4 p_{t-24}$, where p_{t-24} is MCP 24 hours ago. The error filter size has been set to one week ($2M=24*7$), assuming that a given pricing regime exists for at least several days before switching to a different pricing regime, and the stopping criterion α has been set to 0.95.

Since the proposed algorithm incrementally introduces new prediction models into the competition until the stopping criterion is met, intermediate results for both experiments are reported (Table 3). In the first experiment, four regimes are discovered, as introducing the fifth prediction model did not significantly improve accuracy, while in the second experiment, three regimes have been found. The decrease of the MSE by data partitioning is 34% and 16% in the first and second experiments, respectively, relative to the global prediction model. It should be noted that the ratio of MSE between two choices of input variables decreases from 1.41 when only single prediction models are used to 1.10 when the regime discovery algorithm is used. Therefore, the influence of yesterday's price decreases by partitioning the data into different regimes. The role of yesterday's price is to model daily market specifics that could not be described by forecasted load, but rather by conditions on the generation side of the power system. This means that the forecasted load and price relationship are changing over time according to regimes that exist in the market. Still, one can consider that the four regimes accurately reflect the different behaviors since the low error rate is found with fewer inputs. The additional variable of previous price obscures one of the regimes.

Discovered regimes for the two experiments are shown in Fig. 4 together with the time series of forecasted load and MCP in the period from April 1, 1998 till September 30, 1999. As a first observation, the discovered regimes are compact indicating the existence of pricing regimes in the market. The four regimes of experiment 1 are slightly different from the three regimes discovered by experiment 2. While regime 3 with extremely low prices did not repeat in 1999 in experiment 1, the similar regime from experiment 2 reappears in a smaller extent in June of 1999. Also, regime 4 is not distinguished in experiment 2, but remains a part of the most common regime.

Table 4 provides information relevant for the analysis of discovered pricing regimes for both experiments. It explains the size of each regime, range of load in each regime presented with mean \pm one standard deviation of load, regression functions for each regime, accuracy measures of each specialized price predictor as compared to a global predictor, and, finally, the price volatility defined as variance of price differences, $p_t - p_{t-1}$. Design of experiment 1 with MCP predicted as third order polynomial of forecasted load provides more straightforward analysis of change in the market behavior, and therefore we emphasize these results further in text. As mentioned earlier, using only forecasted load for price prediction allows visualization of the obtained regimes. Therefore, Fig. 5 plots the relationship between forecasted load and MCP for the four discovered regimes of experiment 1.

Notice regimes 1 and 2 have similar load-price relationship, with regime 1 being slightly more expensive; however, the volatility of price, defined as the price variance, in regime 1 is significantly larger than for regime 2. Since the loads in both regimes are comparable and there is no indication of significant system changes in these periods, regime 1 might indicate the period of exercising the market power with large suppliers possibly withholding a portion of generation and causing a price increase by shift of supply curve to the left. Regime 3 appearing during the nine weeks of May and June of 1998 is characterized by extremely low price and low volatility apparently caused by high hydro generation and relatively low load. Extremely high prices and high price volatility with a relatively low load characterize regime 4, which appeared during the five weeks of October and November in 1998, one week at the end of December of 1998, and the last week of September of 1999. We speculate this was caused by scheduled maintenance and lower hydro availability but a definitive answer depends on data not publicly available.

Regimes 1 and 2 in experiment 2 have seemingly similar predictors with similar volatility and significantly higher load in regime 1. Still, the dependence of yesterday's price is much lower in regime 2 ($\beta_4=0.18$) than in regime 1 ($\beta_4=0.41$), and further analysis is needed to determine if this is another evidence of market inefficiencies or just the consequence of highly nonlinear price-load relationship at high loads. Regime 3 resembles regime 3 from experiment 1, and corresponds to the period of extremely low prices and price volatility, with a prediction function significantly different from the other two regimes.

Finally, an experiment has been performed to check if inclusion of two dummy variables to indicate peak-hours and weekdays can further improve the price prediction of the regime discovery algorithm. Improvement over not using the dummy variables is less than 1%, which provides additional evidence in support of a finding that hour and day do not influence price, given the forecasted load.

IV. CONCLUSIONS

This paper has reported on characterizing recent price behavior in the California electricity market. An iterative algorithm was introduced that could identify different regimes or characteristic behavior in the data. There are no restrictions on the allowable models in the algorithm although models that require extensive training time or other undesirable characteristics may slow convergence to an unacceptable rate. Our analysis shows that several regimes existed during the time period of our analysis. This information could provide valuable insight to both generation companies and regulators, particularly if supplemented by correlation with data that was not available at the time of market clearing. Further analysis of the identified regimes might identify where market regulations or system infrastructure improvements are needed. In addition, the analysis here has focused on unconstrained prices but price behavior arising from transmission system limits is certainly of great importance. Subsequent work will focus on developments in these areas.

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VI. BIOGRAPHIES

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Captions for the tables

Table 1. Prediction variables with correlations. Load corresponds to a one-day ahead forecasted load known prior to trading.

Table 2. Coefficient of determination of global linear model and neural network model for different variables sets

Table 3. Evolution of prediction accuracy by introducing new competing models

Table 4. Analysis of discovered regimes: size of each regime; load ranges [GWh] expressed as mean \pm one standard deviation; MCP [\$/MWh] prediction functions; R^2 for specialized and global predictors on each regime; and price volatility obtained by (a) experiment 1, (b) experiment 2,

Captions for the graphics

Figure 1. Regime discovery algorithm and Competition procedure.

Figure 2. Mean errors \pm one standard deviation for (a) 24 hours of price prediction, (b) 7 days of price prediction, (c) 24 hours of load forecasting, (d) 7 days of load forecasting.

Figure 3. An example of the sample supply-demand curve of California's electricity market for Jan 25, 1999, 6PM.

Figure 4. (a) Time series of forecasted loads, (b) time series of price, (c) four regimes discovered in Experiment 1, (d) three regimes discovered in experiment 2.

Figure 5. Plot of MCP vs. forecasted load for four regimes discovered in the experiment 1.

Table 1.

Variable	Description	Correlation with price
X ₁	Load [GWh]	0.70
X ₂	Load ²	0.74
X ₃	Load ³	0.77
X ₄	Load _t - Load _{t-1}	0.00
X ₅	Load _t - min_load_for_that_day	0.65
X ₆	Load _t - max_load_for_that_day	-0.51
X ₇	Yesterday's daily average load	0.42
X ₈	Yesterday's load at the same hour	0.64
X ₉	7 days ago daily average load	0.29
X ₁₀	7 days ago load at same hour	0.54
X ₁₁	Yesterday's daily average price	0.56
X ₁₂	Yesterday's price at the same hour	0.81
X ₁₃	7 days ago daily average price	0.31
X ₁₄	7 days ago price at the same hour	0.50

Table 2.

Features used	R ²	
	Linear	NN
X ₁ -X ₁₄	0.79	0.79
X ₁ , X ₂ , X ₃ , X ₁₂	0.77	0.78
X ₁ , X ₂ , X ₁₂	0.74	0.78
X ₁ , X ₂ , X ₃	0.68	0.67
X ₁ , X ₂	0.60	0.68
X ₁₂	0.65	0.65

Table 3.

Number of Regimes	Experiment 1 {X ₁ , X ₂ , X ₃ }		Experiment 2 {X ₁ , X ₂ , X ₃ , X ₁₂ }	
	MSE	R ²	MSE	R ²
All data	93.9	0.68	66.5	0.77
2	76.8	0.74	59.5	0.79
3	65.2	0.78	55.8	0.81
4	61.6	0.79	53.7	0.81
5	61.5	0.79	-	-

Table 4.

(a)

Regime	Size	Load (mean ± std)	MCP, p_t , prediction function	Price prediction R ²		Price Volatility
				Local	Global	
1	5707	26,4 ± 5,8	$-353 + 43I_t - 1.6I_t^2 + 0.021I_t^3$	0.79	0.76	42
2	4630	26,1 ± 5,2	$-285 + 33I_t - 1.2I_t^2 + 0.015I_t^3$	0.81	0.75	19
3	1452	23,9 ± 4,1	$49 - 7I_t + 0.3I_t^2 + 0.004I_t^3$	0.72	-0.49	10
4	1191	24,5 ± 4,2	$-117 + 18I_t - 0.8I_t^2 + 0.012I_t^3$	0.48	-0.03	57

(b)

Regime	Size	Load (mean ± std)	MCP, p_t , prediction function	Price prediction R ²		Price Volatility
				Local	Global	
1	6054	25,3 ± 5,0	$-220 + 27I_t - 1.0I_t^2 + 0.013I_t^3 + 0.41p_{t-24}$	0.75	0.73	35
2	4497	27,3 ± 6,0	$-297 + 35I_t - 1.3I_t^2 + 0.016I_t^3 + 0.18p_{t-24}$	0.83	0.80	40
3	2402	24,3 ± 4,3	$-6 - 0.2I_t + 0.03I_t^2 + 0.00I_t^3 + 0.46p_{t-24}$	0.84	0.69	13

REGIME DISCOVERY ALGORITHM

- Learn a single regression model on global data set S_0 and calculate its MSE, mse_1 .
- Split S_0 into $\mathbb{S}_2^0 = \{S_{1,2}^0, S_{2,2}^0\}$, where $S_{1,2}^0$ and $S_{2,2}^0$ are disjoint and of equal size.
- Modify \mathbb{S}_2^0 through the *competition procedure* (see below) and calculate composite error mse_2 achieved by the obtained partition $\mathbb{S}_2 = \{S_{1,2}, S_{2,2}\}$.
- Terminate the algorithm if the ratio mse_2/mse_1 is larger than α . No regimes are discovered.
- $L=2$.

repeat

for $i = 1$ to L

- Split $S_{i,L}$ into two disjoint subsets of the same sizes, $S_{i,L}'$ and $S_{i,L}''$, to obtain the initial partition $\mathbb{S}_{L+1,i}^0 = \{S_{i,L}/S_{i,L}, S_{i,L}', S_{i,L}''\}$.
- Modify $\mathbb{S}_{L+1,i}^0$ through the *competition procedure* (Fig. 1b.) and calculate composite error $mse_{L+1,i}$ achieved by the obtained partition $\mathbb{S}_{L+1,i} = \{S_{i,L+1,i}, \dots, S_{L+1,L+1,i}\}$.

end

- Out of L partitions, $\mathbb{S}_{L+1,i}$, $i = 1, \dots, L$, choose the partition achieving the smallest composite error, $mse_{L+1,i}$, to represent the new partition, $\mathbb{S}_{L+1} = \{S_{1,L+1}, \dots, S_{L+1,L+1}\}$, and save its composite error as mse_{L+1} .
- $L \leftarrow L + 1$

until the ratio mse_L/mse_{L-1} is smaller than α , $\alpha \in (0,1)$

Output: $L-1$ 'regimes' defined by partition \mathbb{S}_{L-1} and their corresponding models.

COMPETITION PROCEDURE

Start from the initial partition into L subsets \mathbb{S}_L^0 and setting $n=0$.

repeat

- Learn L models on L subsets of the partition \mathbb{S}_L^n , S_i , $i=1, \dots, L$.
- Filter squared or absolute errors of all models on whole time series with moving average filter of size $2M+1$.
- If the model $M_i(\mathbf{x}, \boldsymbol{\beta})$ gives the smallest filtered error on (\mathbf{x}_b, y_b) , assign this point to subset S_i . Form a new partition, \mathbb{S}_L^{n+1} , by reassigning all data in this way.
- $n \leftarrow n+1$

until less than $p\%$ of data points change subsets.

Figure 1

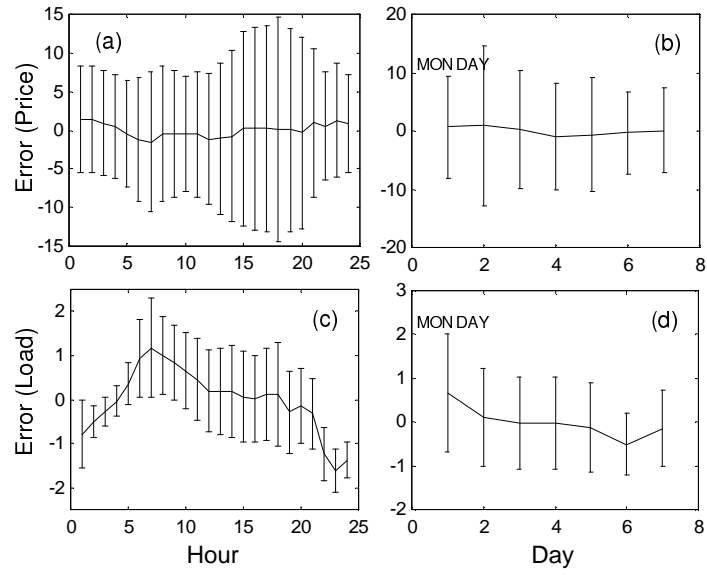


Figure 2.

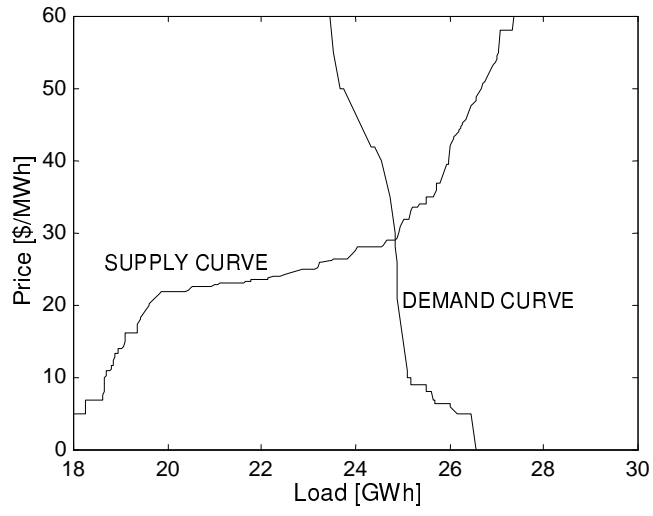


Figure 3.

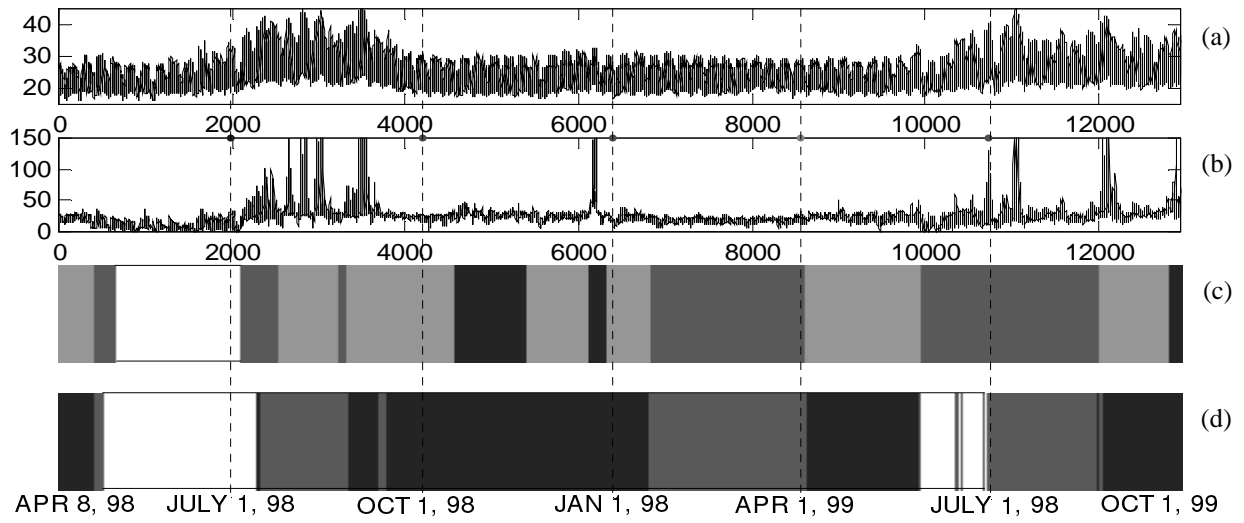


Figure 4.

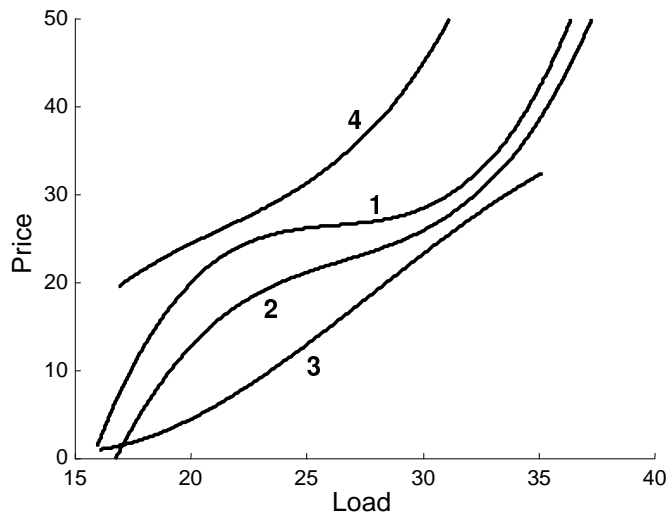


Figure 5.